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HAMILTONIAN FORMULATION OF THE SUPERMEMBRANE

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The hamiltonian formulation of the supermembrane theory in eleven dimensions is given. The covariant split of the first and second class constraints is exhibited, and their Dirac brackets are computed. Gauge conditions are imposed in such a way that the reparametrizations of the membrane with divergence-free 2-vectors are unfixed.

1. Introduction

A supermembrane action in eleven-dimensional spacetime with a nontrivial fermionic symmetry and manifest spacetime supersymmetry has been recently constructed [1]. A consistent truncation of this action on a circle has been shown to yield the action for the type IIA superstring coupled to the vectorlike $N=2$ supergravity in ten dimensions [2]. Furthermore, it has been recently shown that the vacuum energy of the semiclassically quantized fluctuations around a topologically stabilized toroidal background is vanishing [3]. Although these developments are encouraging in quest for finding massless states in the spectrum of the supermembrane, several questions remain unanswered. In particular, not much is known about the infinite dimensional rigid symmetries of the supermembrane which are analogous to the super-Virasoro symmetries of the superstring. These symmetries are naturally expected to play an important role in the understanding of the spectrum of the supermembrane.

A natural framework for studying the Virasoro-like symmetries of an extended object is the hamiltonian formalism. In string theory, the first class constraint given by the traceless energy-momentum tensor, $T_{++}=0$, has the classical Poisson bracket, $\{T_{++}, T_{++}\} = T_{++}$, with no *field dependent* "structure constants". The Fourier expansion of $T_{++} = \Sigma_n L_n \mathrm{e}^{in\sigma}$ defines the Virasoro generators, L_n, which obey the usual Virasoro algebra.

In membrane theory, the first class constraints generate the reparametrization of the three-dimensional world-volume swept by the membrane. Unlike in the string

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case, the Poisson bracket of two first class constraints which generate time reparametrizations involve the field dependent factor $\partial_a X^\mu \partial_b X_\mu$, where $a = 1,2$ labels the spacelike coordinates of the world-volume, and X^μ are the membrane coordinates [4]. Therefore, the search for Virasoro-like symmetries will not only involve harmonic expansions in the two-dimensional membrane parameter space, but also finding an appropriate method to treat the field dependent "structure constants".

The main goal of this paper is to find the explicit form of the full constraint algebra, and the super-Poincaré generators. We show that the theory allows a natural covariant (in the sense of the target space) split of the first and second class constraints (the latter are those which do not generate any symmetries). We impose gauge conditions which respect the reparametrization of the membrane by divergence-free vectors. These transformations have a unit jacobian, and are the analogues of the constant σ-translations of the closed superstring. We perform the reduction of the degrees of freedom down to 8 bosonic X-variables, 8 conjugate momentum variables, and 16 fermionic variables.

The results of this paper should provide a convenient starting point for the quantization of the supermembrane, and the analysis of its spectrum.

This paper is organized as follows. In sect. 2, we give the supermembrane action and its invariances. In sect. 3, the hamiltonian formalism is set up, and the primary and secondary constraints are found. In sect. 4, we separate the first and second class constraints in a covariant fashion, and we calculate their Dirac brackets. In sect. 5, we fix all the symmetries of the supermembrane but the ones which are generated by divergence-free 2-vectors, as mentioned above. In the same section, we also give the Dirac brackets of the transverse degrees of freedom and their equations of motion. Sect. 6 is devoted to conclusions. Our conventions are given in the appendix.

2. The supermembrane action and its symmetries

The action for the closed supermembrane in flat eleven-dimensional spacetime is [1]

$$I = \int \mathrm{d}^3\xi \left[-\tfrac{1}{2}\sqrt{-g}\, g^{ij} \Pi_i^\mu \Pi_{j\mu} + \tfrac{1}{2}\sqrt{-g} - \varepsilon^{ijk} \Pi_i^A \Pi_j^B \Pi_k^C B_{CBA} \right], \tag{2.1}$$

where $\Pi_i^A = (\Pi_i^\mu, \Pi_i^\alpha)$ with

$$\Pi_i^\mu = \partial_i X^\mu - i\bar{\psi}\Gamma^\mu \partial_i \psi \,, \tag{2.2}$$

$$\Pi_i^\alpha = \partial_i \psi^\alpha \,, \tag{2.3}$$

and the membrane tension is set equal to unity. $\xi^i = (\tau, \sigma, \rho)$ $(i = 1,2,3)$ are the coordinates, and g_{ij} is the metric of the world-volume, ψ^α is a 32-component

Majorana spinor. (X^{μ}, ψ^{α}) are the coordinates of the eleven-dimensional superspace. The super 3-form B is such that $\mathrm{d}B = H$, with all components of H vanishing except $H_{\mu\nu\alpha\beta} = -i/3(\Gamma_{\mu\nu})_{\alpha\beta}$. Solving for B, one finds

$$B_{\mu\nu\rho} = 0, \qquad B_{\mu\nu\alpha} = -\tfrac{1}{6}i\left(\Gamma_{\mu\nu}\psi\right)_{\alpha}, \tag{2.4}$$

$$B_{\mu\alpha\beta} = -\tfrac{1}{6}\left(\Gamma_{\mu\nu}\psi\right)_{(\alpha}\left(\Gamma^{\nu}\psi\right)_{\beta)}, \tag{2.5}$$

$$B_{\alpha\beta\gamma} = -\tfrac{1}{6}i\left(\Gamma_{\mu\nu}\psi\right)_{(\alpha}\left(\Gamma^{\mu}\psi\right)_{\beta}\left(\Gamma^{\nu}\psi\right)_{\gamma)}, \tag{2.6}$$

where $(\Gamma\psi)^{\mu}_{\alpha} = \Gamma^{\mu}_{\alpha\beta}\psi^{\beta}$. (For our conventions, see the appendix.) Substituting (2.4)–(2.6) into (2.1), one obtains

$$I = -\tfrac{1}{2}\int \mathrm{d}^3\xi\, \Big[\sqrt{-g}\, g^{ij}\Pi^{\mu}_{i}\Pi_{j\mu} - \sqrt{-g} + i\varepsilon^{ijk}\bar{\psi}\Gamma_{\mu\nu}\,\partial_i\psi$$

$$\times\left(\Pi^{\mu}_{j}\Pi^{\nu}_{k} + i\Pi^{\mu}_{j}\bar{\psi}\Gamma^{\nu}\partial_k\psi - \tfrac{1}{3}\bar{\psi}\Gamma^{\mu}\partial_j\psi\,\bar{\psi}\Gamma^{\nu}\partial_k\psi\right)\Big]. \tag{2.7}$$

The action (2.7) is invariant under the following fermionic transformations [1]

$$\delta X^{\mu} = i\bar{\psi}\Gamma^{\mu}(1+\Gamma)\kappa - i\bar{\psi}\Gamma^{\mu}\varepsilon, \tag{2.8}$$

$$\delta\psi = (1+\Gamma)\kappa + \varepsilon, \tag{2.9}$$

$$\delta\left(\sqrt{-g}\, g^{ij}\right) = 2i\bar{\kappa}(1+\Gamma)\Gamma_{\mu\nu}\,\partial_n\psi\, g^{n(i}\varepsilon^{j)kl}\Pi^{\mu}_{k}\Pi^{\nu}_{l} + \frac{2i}{3\sqrt{-g}}\bar{\kappa}\Gamma^{\mu}\partial_l\psi\,\Pi^{l}_{\mu}\varepsilon^{mn(i}\varepsilon^{j)pq}$$

$$\times\left(\Pi^{\nu}_{m}\Pi_{p\nu}\Pi^{\lambda}_{n}\Pi_{q\lambda} + \Pi^{\nu}_{m}\Pi_{p\nu}g_{nq} + g_{mp}g_{nq}\right), \tag{2.10}$$

where $\kappa = \kappa(\tau, \sigma, \rho)$ is the parameter of the *local* fermionic transformation, and ε is the constant parameter of *rigid* supersymmetry transformations. κ and ε are 32-component Majorana spinors and world-volume scalars [5]. The function Γ is defined by

$$\Gamma = \frac{1}{6\sqrt{-g}}\varepsilon^{ijk}\Pi^{\mu}_{i}\Pi^{\nu}_{j}\Pi^{\rho}_{k}\Gamma_{\mu\nu\rho}. \tag{2.11}$$

On-shell Γ satisfies the relation $\Gamma^2 = 1$, and therefore $\frac{1}{2}(1 \pm \Gamma)$ become projection operators.

The action (2.7) is also invariant under the bosonic transformations given by

$$\delta X^{\mu} = \eta^{i}\,\partial_i X^{\mu} + l^{\mu}_{\nu}X^{\nu} + a^{\mu}, \tag{2.12}$$

$$\delta\psi = \eta^{i}\,\partial_i\psi + \tfrac{1}{4}l_{\mu\nu}\Gamma^{\mu\nu}\psi, \tag{2.13}$$

$$\delta\left(\sqrt{-g}\, g^{ij}\right) = \partial_k\left(\sqrt{-g}\, g^{ij}\eta^{k}\right) - 2\sqrt{-g}\, g^{k(i}\partial_k\eta^{j)}, \tag{2.14}$$

where $\eta = \eta(\tau, \sigma, \rho)$ is the parameter of the general coordinate transformations (i.e. reparametrizations) of the world-volume, and $(l_{\mu\nu} = -l_{\nu\mu}, a^{\mu})$ are the constant parameters of the $d = 11$ *rigid* Poincaré transformations.

The algebra of the κ-transformations closes on-shell. For a detailed discussion of this and several other properties of the supermembrane theory, we refer the reader to the second reference in [1].

3. The covariant hamiltonian formalism

In the hamiltonian formulation of reparametrization invariant systems, it is convenient to parametrize the metric in terms of a shift vector N^a, and a lapse function N as follows (see for example ref. [6]).

$$g_{00} = -N^2 + \gamma_{ab}N^aN^b, \qquad g_{0a} = g_{a0} = \gamma_{ab}N^b,$$
$$g_{ab} = \gamma_{ab}, \qquad \sqrt{-g} = N\sqrt{\gamma}. \tag{3.1}$$
$$g^{00} = -N^{-2}, \qquad g^{0a} = g^{a0} = N^aN^{-2},$$
$$g^{ab} = \gamma^{ab} - N^aN^bN^{-2}. \tag{3.2}$$

Here, γ_{ab} $(a = 1,2)$ is a 2-metric, $\gamma^{ab}\gamma_{bc} = \delta^a_c$, and all variables depend on τ, σ and ρ. In terms of these variables the action (2.7) is readily found to be

$$I = \int d^3\xi \left[\tfrac{1}{2}\sqrt{\gamma}N^{-1}\Pi^{\mu}_0\Pi_{0\mu} - \sqrt{\gamma}N^aN^{-1}\Pi^{\mu}_0\Pi_{a\mu} - \tfrac{1}{2}\sqrt{\gamma}\left(\gamma^{ab} - N^aN^bN^{-1}\right)\Pi^{\mu}_a\Pi_{b\mu}\right.$$
$$\left. + \tfrac{1}{2}\sqrt{\gamma}N + 3\varepsilon^{ab}\Pi^A_0\Pi^B_a\Pi^C_bB_{CBA}\right], \tag{3.3}$$

where $\varepsilon^{12} = -\varepsilon^{21} = 1$.

The canonical variables are $(X^{\mu}, \psi^{\alpha}, N, N^a, \gamma^{ab})$ and their conjugate momenta, $(P_{\mu}, P_{\alpha}, \Pi, \Pi_a, \Pi_{ab})$, which are given by

$$P_{\mu} := \frac{\partial\mathscr{L}}{\partial\dot{X}^{\mu}} = \sqrt{\gamma}N^{-1}\Pi_{0\mu} - \sqrt{\gamma}N^{-1}N^a\Pi_{a\mu} + S_{\mu}, \tag{3.4}$$

$$P_{\alpha} := -\frac{\partial\mathscr{L}}{\partial\dot{\psi}^{\alpha}} = -i(\bar{\psi}\Gamma^{\mu})_{\alpha}P_{\mu} - S_{\alpha}, \tag{3.5}$$

$$\Pi := \frac{\partial\mathscr{L}}{\partial\dot{N}} = 0, \tag{3.6}$$

$$\Pi_a := \frac{\partial\mathscr{L}}{\partial\dot{N}^a} = 0, \tag{3.7}$$

$$\Pi_{ab} := \frac{\partial\mathscr{L}}{\partial\dot{\gamma}^{ab}} = 0, \tag{3.8}$$

where

$$S_\mu := 3\varepsilon^{ab}\Pi_a^A\Pi_b^B B_{BA\mu}$$
$$= -i\varepsilon^{ab}\bar{\psi}\Gamma_{\mu\nu}\,\partial_a\psi\left(\Pi_b^\nu + \tfrac{1}{2}i\bar{\psi}\Gamma^\nu\partial_b\psi\right), \tag{3.9}$$

$$S_\alpha := 3\varepsilon^{ab}\Pi_a^A\Pi_b^B B_{BA\alpha}$$
$$= \tfrac{1}{2}i\varepsilon^{ab}\left(\Gamma_{\mu\nu}\psi\right)_\alpha\left(\Pi_a^\mu\Pi_b^\nu + i\Pi_a^\mu\bar{\psi}\Gamma^\nu\partial_b\psi + \tfrac{1}{3}\bar{\psi}\Gamma^\mu\partial_a\psi\,\bar{\psi}\Gamma^\nu\partial_b\psi\right)$$
$$- \tfrac{1}{2}\varepsilon^{ab}\left(\Gamma^\nu\psi\right)_\alpha\bar{\psi}\Gamma_{\mu\nu}\,\partial_a\psi\left(\Pi_b^\mu - \tfrac{2}{3}i\bar{\psi}\Gamma^\mu\partial_b\psi\right). \tag{3.10}$$

We can solve for the velocity $\dot{X}^\mu$ from (3.4), but we cannot solve for the velocities $\dot{\psi}^\alpha$, $\dot{N}$, $\dot{N}^a$ and $\dot{\gamma}^{ab}$ from (3.5)–(3.8). Therefore we have the *primary constraints* [7, 6]

$$F_\alpha := P_\alpha + i\left(\bar{\psi}\Gamma^\mu\right)_\alpha P_\mu + S_\alpha \approx 0, \tag{3.11}$$

$$\Omega := \Pi \approx 0, \tag{3.12}$$

$$\Omega_a := \Pi_a \approx 0, \tag{3.13}$$

$$\Omega_{ab} := \Pi_{ab} \approx 0. \tag{3.14}$$

These constraints are "weakly zero", meaning that they may have nonvanishing Poisson brackets with some canonical variables [7, 6]. We must now require that the constraints (3.11)–(3.14) are maintained in time, i.e. their Poisson brackets with the hamiltonian is weakly zero. The hamiltonian is given by

$$H := \int \mathrm{d}\sigma\,\mathrm{d}\rho\left(P_\mu\dot{X}^\mu + P_\alpha\dot{\psi}^\alpha - \mathscr{L}\right) + \Sigma^\alpha F_\alpha + \Sigma\Omega + \Sigma^a\Omega_a + \Sigma^{ab}\Omega_{ab}$$
$$= \int \mathrm{d}\sigma\,\mathrm{d}\rho\left[\frac{N}{2\sqrt{\gamma}}\left(P_\mu - S_\mu\right)\left(P^\mu - S^\mu\right) + \tfrac{1}{2}N\sqrt{\gamma}\,\gamma^{ab}\Pi_a^\mu\Pi_{b\mu} - \tfrac{1}{2}N\sqrt{\gamma}\right.$$
$$\left. + N^a\Pi_a^\mu\left(P_\mu - S_\mu\right) + \Sigma^\alpha F_\alpha + \Sigma\Omega + \Sigma^a\Omega_a + \Sigma^{ab}\Omega_{ab}\right], \tag{3.15}$$

where the Σ's are Lagrange multipliers. The Poisson brackets of two arbitrary functions, A and B, of the canonical variables is defined by

$$\{A, B\} := \int \mathrm{d}\sigma\,\mathrm{d}\rho \Bigg[(-1)^{A+1} \frac{\delta A}{\delta \psi^\alpha} \frac{\delta B}{\delta P_\alpha} + (-1)^{AB+B} \frac{\delta B}{\delta \psi^\alpha} \frac{\delta A}{\delta P_\alpha} + \left(\frac{\delta A}{\delta X^\mu} \frac{\delta B}{\delta P_\mu} + \frac{\delta A}{\delta N} \frac{\delta B}{\delta \Pi} + \frac{\delta A}{\delta N^a} \frac{\delta B}{\delta \Pi_a} + \frac{\delta A}{\delta \gamma^{ab}} \frac{\delta B}{\delta \Pi_{ab}} - (-1)^{AB} (A \leftrightarrow B) \right) \Bigg], \tag{3.16}$$

where the grading $A = 0$ for bosons, and $A = 1$ for fermions. In particular,

$$\{X^\mu(\xi), P_\nu(\xi')\} = \delta^\mu_\nu \delta^2(\xi - \xi'), \qquad \{\psi^\alpha(\xi), P_\beta(\xi')\} = \delta^\alpha_\beta \delta^2(\xi - \xi'),$$

$$\{N(\xi), \Pi(\xi')\} = \delta^2(\xi - \xi'), \qquad \{N^a(\xi), \Pi_b(\xi')\} = \delta^a_b \delta^2(\xi - \xi'),$$

$$\{\gamma^{ab}(\xi), \Pi_{cd}(\xi')\} = \tfrac{1}{2}\left(\delta^a_c \delta^b_d + \delta^b_c \delta^a_d\right) \delta^2(\xi - \xi'), \tag{3.17}$$

where the brackets are evaluated at equal times, and therefore ξ stands for σ and ρ.

Requiring that the hamiltonian, (3.15), has weakly vanishing Poisson brackets with the primary constraints Ω, Ω_a and Ω_{ab}, defined in (3.12)–(3.14), one readily finds the *secondary constraints*,

$$\varphi := \tfrac{1}{2}(P_\mu - S_\mu)(P^\mu - S^\mu) + \tfrac{1}{2}\gamma\gamma^{ab}\Pi^\mu_a \Pi_{b\mu} - \tfrac{1}{2}\gamma \approx 0, \tag{3.18}$$

$$\varphi_a := \Pi^\mu_a (P_\mu - S_\mu) \approx 0, \tag{3.19}$$

$$\varphi_{ab} := \Pi^\mu_a \Pi_{b\mu} - \gamma_{ab} \approx 0. \tag{3.20}$$

The Poisson bracket $\{F_\alpha, H\}$ is far more complicated. We find that it does not lead to any new constraints, but it enables us to solve for the Lagrange multiplier $\frac{1}{2}(1 - \Gamma)^\alpha_\beta \Sigma^\beta \equiv \Sigma^\alpha_-$. Multiplying the secondary constraints φ, φ_a and φ_{ab} with the Lagrange multipliers Λ, Λ_a and Λ_{ab}, respectively, and adding them to the hamiltonian, (3.15), we obtain the total hamiltonian

$$H' = \int \mathrm{d}\sigma\,\mathrm{d}\rho \left[\left(\left(\frac{N}{\sqrt{\gamma}} + \Lambda \right) \varphi + (N^a + \Lambda^a)\varphi_a + \Sigma\Omega + \Sigma^a \Omega_a + \Sigma_{+\alpha} F^\alpha_+ \right) + \left(\Lambda^{ab}\varphi_{ab} + \Sigma^{ab}\Omega_{ab} + \Sigma_{-\alpha} F^\alpha_- \right) \right]. \tag{3.21}$$

One should now verify that the secondary constraints (3.18)–(3.20) are also maintained in time, i.e. their Poisson bracket with H' vanishes weakly. We find that $\{\Omega_{ab}, H'\} \approx 0$ enables us to solve for the Lagrange multiplier Σ_{ab}, while the requirement that $\{\varphi_{ab}, H'\} \approx 0$ enables us to determine the Lagrange multiplier Λ_{ab}. The Poisson brackets of the hamiltonian H' with the remaining constraints are weakly zero. Thus, there are no new (tertiary) constraints and *the constraints (3.11)–(3.14) and (3.18)–(3.20) form a complete set*. In the next section we shall separate them into first and second class constraints and calculate their Dirac brackets [7].

Note that in the hamiltonian (3.21), the Lagrange multipliers Λ, Λ_a, Σ, Σ_a, and $\Sigma_{+\alpha}$ are still undetermined. This is a consequence of the reparametrization and fermionic invariances of the theory.

4. The first and second class constraints and their Dirac brackets

The first class constraints are those which generate infinitesimal transformations that are the symmetries of the theory. Denoting the first class constraints by Φ_R, their Poisson brackets are weakly zero, $\{\Phi_R, \Phi_S\} \approx 0$, which means that the right-hand side is proportional to constraints. Any other constraint is second class. Denoting the second class constraints by χ_r, their Poisson bracket is of the form

$$\{\chi_r, \chi_s\} \approx C_{rs}, \tag{4.1}$$

where " $\approx$ " means "modulo constraints", and C_{rs} is an even dimensional nonsingular matrix which is a function of the canonical variables. Our task is to compute C_{rs}. We can then compute the Dirac brackets of any two canonical variables, A and B, which is defined as [7]

$$\{A, B\}^* = \{A, B\} - \int \mathrm{d}\xi' \mathrm{d}\xi'' \{A, \chi_r(\xi')\} C^{-1rs}(\xi', \xi'')\{\chi_s(\xi''), B\}. \tag{4.2}$$

Note that the Dirac bracket is defined such that $\{A, \chi_r\}^* \approx 0$, for any A. In fact, it is this property which allows us to take C_{rs}^{-1} to be modulo constraints in (4.1) and (4.2).

After a long calculation we find that the second and first class constraints of the theory are the following:

Second class constraints

$$\Omega_{ab} := \Pi_{ab} \approx 0, \tag{4.3}$$

$$\varphi_{ab} := \Pi_a^\mu \Pi_{b\mu} - \gamma_{ab} \approx 0, \tag{4.4}$$

$$\lambda_- := \tfrac{1}{2}(1 - \Gamma)\left(P - iP_\mu \Gamma^\mu \psi + S + 4i\Omega^{ab}\Pi_a^\mu \Gamma_\mu \partial_b \psi\right) \approx 0. \tag{4.5}$$

First class constraints

$$\Omega := \Pi \approx 0, \tag{4.6}$$

$$\Omega_a := \Pi_a \approx 0, \tag{4.7}$$

$$\lambda_+ := \tfrac{1}{2}(1+\Gamma)\left(P - iP_\mu \Gamma^\mu \psi + S + 4i\Omega^{ab}\Pi^\mu_a \Gamma_\mu \partial_b \psi\right) \approx 0, \tag{4.8}$$

$$T := \tfrac{1}{2}(P_\mu - S_\mu)(P^\mu - S^\mu) + \tfrac{1}{2}\gamma\gamma^{ab}\Pi^\mu_a \Pi_{b\mu} - \tfrac{1}{2}\gamma$$
$$+ 2\Omega^{ab}\partial_a \Pi^\mu_b (P_\mu - S_\mu) + i\varepsilon^{ab}\Pi^\mu_a \partial_b \bar{\psi}\Gamma_\mu \lambda \approx 0, \tag{4.9}$$

$$T_a := \Pi^\mu_a (P_\mu - S_\mu) + 2\Omega^{cd}\partial_c \Pi^\mu_d \Pi_{a\mu}$$
$$+ 2\gamma^{bc}\partial_b \Omega_{ca} + \partial_a \bar{\psi}\lambda \approx 0, \tag{4.10}$$

where $\lambda = \lambda_+ + \lambda_-$. The term proportional to the constraint Ω_{ab} in (4.5), and the terms proportional to the constraint λ_+ in (4.9), (4.10) are added for convenience, while the terms proportional to the constraint Ω_{ab} in (4.8) and the terms proportional to the constraints Ω_{ab} and λ_- in (4.9), (4.10) are necessary to render the constraint T, T_a and λ_+ first class. The matrix Γ is defined in (2.11). It can be written in terms of the canonical variables as

$$\Gamma = -\frac{1}{2\gamma}\varepsilon^{ab}(P^\mu - S^\mu)\Pi^\nu_a \Pi^\rho_b \Gamma_{\mu\nu\rho}, \tag{4.11}$$

and it satisfies the relation

$$\Gamma^2 = 1 - 2\gamma^{-1}T + \gamma^{-1}T^a T_a + \text{second class constraints}. \tag{4.12}$$

Note that the fermionic first and second class constraints are separated in an SO(10, 1) covariant manner. This separation deserves some comments. First, one finds that

$$\{F_\alpha(\xi), F_\beta(\xi')\} = \delta^2(\xi - \xi')\Big[-2i(P_\mu - S_\mu)(\Gamma^\mu(1-\Gamma))_{\alpha\beta} + 2i(\Gamma_{\mu\nu})_{\alpha\beta}\gamma^{-1}\varepsilon^{ab}\Pi^\nu_b$$
$$\left(\Pi^\mu_a \varphi - \tfrac{1}{2}\gamma \Pi^\mu_a \gamma^{cd}\varphi_{cd} - (P^\mu - S^\mu)\varphi_a\right)\Big]. \tag{4.13}$$

From this equation it is clear how to separate the first and second class fermionic constraints covariantly. Defining $F_\pm = \frac{1}{2}(1 \pm \Gamma)F$, from (4.13) it readily follows that $\{F_+, F_+\} \approx 0$, while $\{F_-, F_-\}$ is an invertible matrix of rank 16. Thus, F_- are second class constraints, and F_+ are first class constraints provided that they have weakly vanishing Poisson brackets also with the other first class constraints. This last property is easily obtained by adding to F_+ the term proportional to the constraint Ω^{ab}, as in (4.8). Similar features have been encountered before in the hamiltonian formulation of the Green-Schwarz superstring [8].

We now turn to the evaluation of C_{rs}. The only nonvanishing Poisson brackets between the second class constraints, (4.3)–(4.5), are

$$\{\lambda^{\alpha}_{-}(\xi), \lambda^{\beta}_{-}(\xi')\} \approx -2i(P_{\mu}-S_{\mu})(\Gamma^{\mu}(1-\Gamma))^{\alpha\beta}\delta^2(\xi-\xi'), \tag{4.14}$$

$$\{\Omega_{ab}(\xi), \varphi_{cd}(\xi')\} \approx -\tfrac{1}{2}(\gamma_{ac}\gamma_{bd}+\gamma_{bc}\gamma_{ad})\delta^2(\xi-\xi'). \tag{4.15}$$

A simple calculation yields the result,

$$\left[(P_{\mu}-S_{\mu})\Gamma^{\mu}(1-\Gamma)\right]^{-1} \approx \tfrac{1}{4}\gamma^{-1}\left[(P_{\mu}-S_{\mu})\Gamma^{\mu}(1-\Gamma)\right], \tag{4.16}$$

which can be used together with (4.1), (4.2) and (4.14), (4.15) to find the Dirac bracket

$$\begin{aligned}\{A,B\}^* = \{A,B\} &- \int \mathrm{d}\sigma\,\mathrm{d}\rho\,\{A,\Omega_{ab}\}\gamma^{ac}\gamma^{bd}\{\varphi_{cd},B\}\\ &+ \int \mathrm{d}\sigma\,\mathrm{d}\rho\,\{A,\varphi_{ab}\}\gamma^{ac}\gamma^{bd}\{\Omega_{cd},B\}\\ &- \tfrac{1}{4}i\int \mathrm{d}\sigma\,\mathrm{d}\rho\,\{A,\lambda_-\}\gamma^{-1}(P_{\mu}-S_{\mu})\Gamma^{\mu}\{\lambda_-,B\}.\end{aligned} \tag{4.17}$$

The second class constraints, (4.3)–(4.5), have weakly vanishing Dirac brackets with *any* canonical variable. Therefore they can be set *strongly* equal to zero. Thus, we are left with the task of evaluating the Dirac brackets of the first class constraints, (4.6)–(4.10). On the right-hand side of (4.17), after computing the Poisson brackets, any second class constraint encountered can be set strongly equal to zero. Moreover, since the Poisson bracket of a first class constraint with a second class constraint yields combination of constraints, the last three terms in (4.17) are quadratic in constraints, when A and B are first class constraints. However, any quadratic constraint evidently has weakly vanishing Poisson brackets with any canonical variables, and therefore is second class. Thus, in evaluating the Dirac brackets of two first class constraints the last three terms in (4.17) can be set strongly equal to zero.

The only nonvanishing Dirac brackets of the first class constraints (4.6)–(4.10), which we find are the following:

$$\{T(\xi), T(\xi')\}^* = \partial_a\delta^2(\xi-\xi')\left[\left(\gamma\gamma^{ab}T_b\right)(\xi) + \left(\gamma\gamma^{ab}T_b\right)(\xi')\right], \tag{4.18}$$

$$\{T(\xi), T_a(\xi')\}^* = 2\,\partial_a\delta^2(\xi-\xi')T(\xi) + \delta^2(\xi-\xi')\,\partial_a T(\xi), \tag{4.19}$$

$$\{T_a(\xi), T_b(\xi')\}^* = \partial_a\delta^2(\xi-\xi')T_b(\xi) + \partial_b\delta^2(\xi-\xi')T_a(\xi'), \tag{4.20}$$

$$\{\lambda_+(\xi), T_a(\xi')\}^* = \partial_a\delta^2(\xi-\xi')\lambda_+(\xi) + \delta^2(\xi-\xi')\,\partial_a\lambda_+(\xi), \tag{4.21}$$

$$\begin{aligned}\{\lambda_+(\xi), T(\xi')\}^* = {} & i\varepsilon^{ab}(1+\Gamma)\,\partial_a\psi\, T_b\delta^2(\xi-\xi') + \partial_a\delta(\xi-\xi') \\ & \times\left[-\Gamma^{\mu\nu}\lambda_+\left(P_\mu - S_\mu\right)\Pi_{b\nu}\gamma^{ab}\right](\xi) + \delta^2(\xi-\xi') \\ & \times\Big[-\Gamma^{\mu\nu}\partial_a\lambda_+\left(P_\mu - S_\mu\right)\Pi_{b\nu}\gamma^{ab} \\ & -2i\Gamma^{\mu\nu}\lambda_+\left(P_\mu - S_\mu\right)\Pi_{a\nu}\Pi^\rho_b\,\partial_c\bar\psi\Gamma^\rho\partial_d\psi\gamma^{ad}\gamma^{bc} \\ & +\tfrac{1}{2}i\gamma^{-1}\varepsilon^{ab}\varepsilon^{cd}\Gamma_{\rho\sigma}\lambda_+\left(P_\mu - S_\mu\right)\Pi^\rho_c\Pi^\sigma_d\,\partial_a\bar\psi\,\Gamma^\mu\partial_b\psi \\ & +\tfrac{1}{2}i\varepsilon^{ab}\Gamma_\mu\lambda_+\partial_a\bar\psi\,\Gamma^\mu\partial_b\psi + \tfrac{1}{4}i\varepsilon^{ab}\Gamma_{\mu\nu}\lambda_+\partial_a\bar\psi\,\Gamma^{\mu\nu}\partial_b\psi\Big](\xi),\end{aligned} \tag{4.22}$$

$$\begin{aligned}&\{\lambda_{+\alpha}(\xi), \lambda_{+\beta}(\xi')\}^* \\ &= \delta^2(\xi-\xi')\Big\{\left(\Gamma_\mu(1+\Gamma)\right)_{\alpha\beta}\left[\gamma^{-1}(P^\mu - S^\mu)T - \Pi^\mu_a T^a\right] - \left(\Gamma_\mu(1+\Gamma)\right)_{\alpha\beta}\Pi^\mu_b\gamma^{ab} \\ &\qquad \times\left(\partial_a\bar\psi\lambda_+\right) + 2i\lambda_{+(\alpha}\left[(1+\Gamma)\Gamma^\mu\partial_a\psi\right]_{\beta)}\Pi_{b\mu}\gamma^{ab} + \gamma^{-1}\varepsilon^{ab} \\ &\qquad \times\left(\Gamma^\mu(1+\Gamma)\right)_{\alpha\beta}\left(\partial_a\bar\psi\,\Gamma^\nu\lambda_+\right)\left(P_\mu - S_\mu\right)\Pi_{b\nu} - i\gamma^{-1}\varepsilon^{ab} \\ &\qquad \times\left(\Gamma_{\mu\nu\rho}\lambda_+\right)_{(\alpha}\left((1+\Gamma)\Gamma^\rho\partial_a\psi\right)_{\beta)}\left(P^\mu - S^\mu\right)\Pi^\nu_b + \tfrac{1}{2}i\gamma^{-1}\varepsilon^{ab}\varepsilon^{cd} \\ &\qquad \times\left(\Gamma_{\mu\nu\rho}\lambda_+\right)_{(\alpha}\left((1+\Gamma)\Gamma^{\mu\sigma}\partial_a\psi\right)_{\beta)}\Pi_{b\sigma}\Pi^\nu_c\Pi^\rho_d\Big\}(\xi).\end{aligned} \tag{4.23}$$

Clearly T and T_a generate the general coordinate transformations in the three-dimensional world-volume of the supermembrane, and λ_+ the local fermionic transformations. The content of (4.18)–(4.23) can be expressed schematically as

$$\{\delta(\eta_1),\delta(\eta_2)\}=\delta(\eta_3), \tag{4.24}$$

$$\{\delta(\eta),\delta(\kappa)\}=\delta(\kappa')+\delta(\eta'), \tag{4.25}$$

$$\{\delta(\kappa_1),\delta(\kappa_2)\}=\delta(\kappa_3)+\delta(\eta). \tag{4.26}$$

The notation is self-explanatory. The second term on the right-hand side of (4.25) arises because of the admixture of the first class constraint λ_+ in the definition of the first class constraint T, as in (4.9).

The supermembrane theory is also invariant under the rigid $d=11$ super-Poincaré transformations. The generators of these transformations are the momentum P_μ, the Lorentz generators $M_{\mu\nu}$ and the supercharge Q^α which can be computed by standard methods. The result is

$$M_{\mu\nu}=\int \mathrm{d}\sigma\,\mathrm{d}\rho\left(X_\mu P_\nu - X_\nu P_\mu - \tfrac{1}{2}\bar{P}T_{\mu\nu}\psi\right), \tag{4.27}$$

$$\begin{aligned} Q=\int \mathrm{d}\sigma\,\mathrm{d}\rho\,\Big[& iP^\mu\Gamma_\mu\psi + P + \varepsilon^{ab}\Gamma_{\mu\nu}\psi \\ &\times\left(-\tfrac{1}{2}i\Pi^\mu_a\Pi^\nu_b + \tfrac{5}{6}\Pi^\mu_a\bar{\psi}\Gamma^\nu\partial_b\psi\,\bar{\psi}\Gamma\psi + \tfrac{11}{30}i\bar{\psi}\Gamma^\mu\partial_a\psi\,\bar{\psi}\Gamma^\nu\partial_b\psi\right) \\ &-\tfrac{1}{3}\varepsilon^{ab}\Gamma^\nu\psi\bar{\psi}\Gamma_{\mu\nu}\,\partial_b\psi\left(\tfrac{1}{2}\Pi^\mu_a+\tfrac{2}{5}i\bar{\psi}\Gamma^\mu\partial_a\psi\right)\Big]. \end{aligned} \tag{4.28}$$

5. Gauge fixing and the reduction of degrees of freedom

In the hamiltonian formalism gauge fixing is done by imposing new constraints, G_R, such that all the first class constraints, Φ_R, become second class. This means that the gauge conditions must satisfy the relation $\{G_R,\Phi_S\}\not\approx 0$. Consequently, *all* the constraints in the theory become second class, and therefore all of them can be imposed as strong equations, provided that we use Dirac brackets. In this procedure the zero modes, if any, must be handled with care. They correspond to the unfixed (residual) rigid symmetries.

A convenient set of gauge conditions which are similar to the light-cone gauge conditions for the Green-Schwarz superstrings are

$$G_1 := X^+ - P_0^+\tau \approx 0, \qquad G_2 := P^+ - P_0^+ \approx 0, \tag{5.1}$$

$$G := N-1\approx 0, \tag{5.2}$$

$$G^a := N^a \approx 0, \tag{5.3}$$

$$G^\alpha := (\Gamma^+\psi)^\alpha + \tfrac{1}{4}i\gamma^{-1}(P_\mu - S_\mu)(\Gamma^+\Gamma^\mu\lambda_-)^\alpha + \tfrac{1}{8}i\gamma^{-1}(P_\mu - S_\mu)(\Gamma^+\Gamma^\mu\lambda_+)^\alpha, \tag{5.4}$$

where P_0^+ is the constant mode of P^+. The last two terms in (5.4) are added so that the only nonvanishing Poisson bracket of G_α is with the first class constraint $\lambda_{+\alpha}$. In practice this does not complicate matters: Since the second class constraints are set strongly equal to zero after brackets are evaluated, effectively we will be working with the simple gauge condition $\Gamma^+\psi \approx 0$.

The only nonvanishing Poisson brackets of the gauge conditions (5.1)–(5.4), with any other constraints are

$$\{G_1(\xi), T(\xi')\} \approx P_0^+\delta^2(\xi-\xi'), \tag{5.5}$$

$$\{G_2(\xi), T_a(\xi')\} \approx P_0^+\,\partial_a\delta^2(\xi-\xi'), \tag{5.6}$$

$$\{G^\alpha(\xi), \lambda_{+\beta}(\xi')\} \approx \tfrac{1}{2}(\Gamma^+(1+\Gamma))^\alpha_\beta\delta^2(\xi-\xi'), \tag{5.7}$$

$$\{G(\xi), \Omega(\xi')\} \approx \delta^2(\xi-\xi'), \tag{5.8}$$

$$\{G^a(\xi), \Omega_b(\xi')\} \approx \delta^a_b\delta^2(\xi-\xi'). \tag{5.9}$$

From these brackets, the nature of the symmetries fixed by the gauge conditions is rather transparent. Actually not all symmetries are fixed by the gauge conditions, (5.1)–(5.4). This is so because the curl of the first class constraint drops out of (5.6). Therefore *while $\nabla_a T^a$ is second class, $\varepsilon^{ab}\nabla_a T_b$ is still a first class constraint.* From,

$$\int \mathrm{d}\xi'\,\{f\gamma^{-1/2}\varepsilon^{ab}\nabla_a T_b(\xi'), X^\mu(\xi)\} = -\varepsilon^{ab}\gamma^{-1/2}\partial_b f\partial_a X^\mu, \tag{5.10}$$

where $f(\tau,\sigma,\rho)$ is an arbitrary transformation parameter, we see that the first class constraint $\varepsilon^{ab}\nabla_a T_b$ generates the reparametrizations $\delta X^\mu = \eta^i\,\partial_i X^\mu$, with $\eta^0 = 0$ and $\eta^a = -\gamma^{-1/2}\varepsilon^{ab}\,\partial_b f(\tau,\sigma,\rho)$ [9].

Finally, if the membrane is connected but not simply connected, there will be certain global constraints which remain first class. They arise as follows. According to Hodge theorem (see for example ref. [10]) a vector on an arbitrary compact manifold can be decomposed into divergence free, curl free and harmonic parts. Harmonic vectors are curl and divergence free vectors, and there are as many harmonic vectors as the first Betti number of the manifold. Now, a compact Riemann surface of genus g, which is the membrane we are considering here, has the Betti number $2g$. Therefore considering the Hodge decomposition of φ_a we see that there are $2g$ global constraints $\oint T_a\,\mathrm{d}l^a$, where l is any one of the $2g$ noncontractible loops on the membrane. Note that the part proportional to $\nabla_a T^a$ drops out in the integral, so that one is left with the harmonic part of T^a, provided that $\varepsilon^{ab}\nabla_a T_b = 0$.

In summary, given the gauge conditions (5.1)–(5.4), the only first class constraints remaining in the theory are

$$C_1 := \varepsilon^{ab} \nabla_a T_b \approx 0 \,, \tag{5.11}$$

$$C_2 := \oint T_a \mathrm{d} l^{a(w)} \approx 0 \,, \qquad w = 1, \dots, 2g \,. \tag{5.12}$$

Note that (5.12) has the same content as (5.11) for contractible loops. Of course, this situation is also encountered in the lagrangian formulation [3].

At this stage the constraints (4.3)–(4.9), $\nabla_a T^a \approx 0$, and (5.1)–(5.4) are second class, and therefore they can be set strongly equal to zero. Consequently, the total hamiltonian, (3.21), reduces to

$$\begin{aligned} H &= \int \mathrm{d}\sigma\, \mathrm{d}\rho \left(\gamma^{-1/2} \varepsilon^{ab} \partial_b f T_a + \sum_{w=1}^{2g} c_w \oint T_a \mathrm{d} l^{a(w)} \right) \\ &= \int \mathrm{d}\sigma\, \mathrm{d}\rho \left(\gamma^{-1/2} \varepsilon^{ab} \partial_b f + \sum_{w=1}^{2g} c_w \Lambda_{\mathrm{H}}^{a(w)} \right) (P_\mu - S_\mu) \Pi_a^\mu \end{aligned} \tag{5.13}$$

where $f(\tau, \sigma, \rho)$ is an arbitrary function, and the harmonic part of Λ^a denoted by Λ_{H}^a is multiplied by $2g$ arbitrary constant coefficients c_w.

We could consider fixing the diffeomorphisms generated by C_1. This can be achieved, for example, by the gauge conditions $X^1 - \rho \approx 0$ and $P^1|_{\sigma=\sigma_0} \approx 0$. However, in that case the Dirac brackets become enormously complicated. Therefore, we choose not to fix the symmetries generated by C_1. The price one has to pay is the necessity to impose the constraints C_1 and C_2 on the states as physical conditions; $C_1 |\mathrm{phys}\rangle = C_2 |\mathrm{phys}\rangle = 0$.

From the second class constraints (4.3)–(4.5), (4.8) and (5.1)–(5.4), all of which are now set strongly equal to zero, we solve for the unphysical variables,

$$X^+ = P_0^+ \tau \,, \qquad P^+ = P_0^+ \,, \qquad \Gamma^+ \psi = 0 \,, \tag{5.14}$$

$$N^a = 0 \,, \qquad N = 1 \,, \tag{5.15}$$

$$\gamma_{ab} = \partial_a \boldsymbol{X} \cdot \partial_b \boldsymbol{X} \,, \tag{5.16}$$

where boldface indicates the nine-dimensional transverse directions. From $T = 0$ we solve for P^-:

$$P^- = S^- + \frac{1}{2P_0^+} \left[\boldsymbol{P}^2 + \gamma \right] , \tag{5.17}$$

while, from $\nabla_a T^a = 0$ we solve for X^-:

$$X^- = X_0^- + \Delta^{-1}\nabla^a\left[i\bar{\psi}\Gamma^-\partial_a\psi + \frac{1}{P_0^+}\partial_a \boldsymbol{X}\cdot\boldsymbol{P}\right], \tag{5.18}$$

where X_0^- is an integration constant and $\Delta = \nabla^a\partial_a$. There remains the second class constraint $\lambda = 0$, which determines the conjugate momentum P completely,

$$P = iP_\mu \Gamma^\mu \psi - S. \tag{5.19}$$

In summary, the independent canonical variables are $\boldsymbol{X}$, $\boldsymbol{P}$, X_0^-, P_0^+ and $\Gamma^+\Gamma^-\psi$. Imposing the first class constraints (5.11), (5.12) on a physical state in X-space (the wave function) leads to 8 independent X-variables. The Fourier transform of the state then yields a function of 8 independent P-variables. Therefore, the physical phase space actually consists of 8 X-variables, 8 P-variables and 16 fermionic variables P, half of which behave as a conjugate fermionic momenta since $\{\Gamma^+\Gamma^-\psi,\ \Gamma^+\Gamma^-\psi\} \neq 0$.

With the second class constraints given in (4.3)–(4.9), $\nabla_a T^a \approx 0$ and (5.1)–(5.4), we find that for the gauge fixed theory the Dirac bracket (4.2) is given by

$$\begin{aligned}\{A,B\}^* = \{A,B\} - \int \mathrm{d}\sigma\,\mathrm{d}\rho\,\Big[&\{A,\Omega_{ab}\}\gamma^{ac}\gamma^{bd}\{\varphi_{cd},B\} - \{A,T\}(P_0^+)^{-1}\\ &\times\{G_1,B\} + \{A,\nabla_a T^a\}(P_0^+\nabla^c\partial_c)^{-1}\{G_2,B\}\\ &-\{A,\Omega_a\}\gamma^{ab}\{G_b,B\} - \{A,\Omega\}\{G,B\} - \tfrac{1}{8}i\gamma^{-1}\\ &\times\{A,\lambda_-\}(P_\mu - S_\mu)\Gamma^\mu\{\lambda_-,B\} + i\{A,G\}(P_0^+)^{-1}\\ &\times(P_\mu - S_\mu)\Gamma^\mu\{\lambda_+,B\} - (-1)^{AB}(A\leftrightarrow B)\Big].\end{aligned} \tag{5.20}$$

In order to put the basic Dirac brackets into simple form, we make the following field redefinitions

$$\tilde{X}_0^- := X_0^- - P_0^-\tau, \tag{5.21}$$

$$\chi := (P_0^+)^{1/2}\Gamma^+\Gamma^-\psi. \tag{5.22}$$

With these redefinitions, the only nonvanishing Dirac brackets between the canonical variables are

$$\{X^\mu(\xi), P_\nu(\xi')\}^* = \delta_\nu^\mu\delta^2(\xi-\xi'), \qquad \mu,\nu \neq \pm, \tag{5.23}$$

$$\{\tilde{X}_0^-, P_0^+\}^* = -1, \tag{5.24}$$

$$\{\chi^\alpha(\xi), \chi_\beta(\xi')\}^* = i(\Gamma^+)_\beta^\alpha\delta^2(\xi-\xi'). \tag{5.25}$$

The equations of motion for an arbitrary function A of canonical variables and time are given by

$$\dot{A} = \{A, H\} + \frac{\partial A}{\partial \tau} \tag{5.26}$$

$$= \{A, H\}^* + \int d^2\xi' \{A, \varphi(\xi')\}(P_0^+)^{-1}\{G_1(\xi'), H\} + \frac{\partial A}{\partial \tau} \tag{5.27}$$

$$= \left[-\gamma^{-1/2}\varepsilon^{ab}\,\partial_b f + \sum_{w=1}^{2g} c_w \Lambda_{\mathrm{H}}^{a(w)}\right]\partial_a A + \int d^2\xi' \{A, \varphi(\xi')\} + \frac{\partial A}{\partial \tau}. \tag{5.28}$$

It is important to note that in (5.26) the hamiltonian is the one given in (3.21) since the Poisson bracket is used. However, in (5.27), since we have rewritten (5.26) in such a way that we have a Dirac bracket, all the second class constraints can be set strongly equal to zero in the hamiltonian, which therefore reduces to the expression (5.13). Note also that, of all second class constraints, only G_1 survives in (5.27), due to the fact that it is the only constraint which contains an explicit time dependence.

From (5.28) we then find the following field equations:

$$\dot{\boldsymbol{X}} = \Lambda^a\,\partial_a \boldsymbol{X} + \boldsymbol{P}, \tag{5.29}$$

$$\dot{\boldsymbol{P}} = \Lambda^a\,\partial_a \boldsymbol{P} + \partial_a\left(\gamma\gamma^{ab}\partial_b \boldsymbol{X}\right) + \varepsilon^{ab}\,\partial_a\bar{\chi}\Gamma^-\boldsymbol{\Gamma}\partial_b\chi, \tag{5.30}$$

$$\dot{\tilde{X}}_0^- = 0, \tag{5.31}$$

$$\dot{P}_0^+ = 0, \tag{5.32}$$

$$\dot{\chi} = \Lambda^a\,\partial_a\chi - i\varepsilon^{ab}\,\partial_a \boldsymbol{X}\cdot\boldsymbol{\Gamma}\partial_b\chi, \tag{5.33}$$

where $\boldsymbol{\Gamma}$ denotes the Γ-matrices in 9 transverse dimensions, and $\nabla_a \Lambda^a = 0$, whose solution is

$$\Lambda^a = -\gamma^{-1/2}\varepsilon^{ab}\,\partial_b f + \sum_{w=1}^{2g} c_w \Lambda_{\mathrm{H}}^{a(w)}. \tag{5.34}$$

Here, Λ_{H}^a is the harmonic part of Λ^a.

6. Conclusions

In this paper we have computed the algebra of constraints in the eleven-dimensional supermembrane theory. We have shown explicitly how to separate the first and second class fermionic constraints in a covariant fashion. We have imposed gauge conditions which fix all the symmetries of the theory (i.e. reparametrization and the fermionic symmetry) except the reparametrizations $\delta\xi^a$ obeying the relation,

$$\delta\xi^0 = 0, \qquad \nabla_a(\delta\xi^a) = 0. \tag{6.1}$$

It was convenient to preserve this residual symmetry of the hamiltonian (fixing them leads to exceedingly complicated Dirac brackets), and to impose the constraints implied by (6.1) on states. We have performed the reduction of the degrees of freedom down to 8 X-variables, 8 associated canonical momenta, and 16 fermionic variables. We have computed the Dirac brackets of the transverse degrees of freedom and their equations of motion.

The results of this paper should provide a convenient starting point for the quantization of the supermembrane, as mentioned in the introduction. Solving the theory exactly at the quantum level seems to be rather difficult, at present, mainly due to the nonlinearities in the field equations and in the constraint algebra. Nonetheless, one can utilize semiclassical quantization methods, as in ref. [3], to probe the theory. We believe that already at that level the theory is likely to exhibit rich structures.

One of the basic problems involved in the quantization is the normal ordering problem. Suppose that we work with the gauge unfixed theory. We must show that with a certain normal ordering scheme the constraint algebra and the super-Poincaré algebra are maintained at the quantum level. (In string theory, the latter symmetry would be manifest in the covariant formalism but not in the supermembrane theory due to the nonlinearities in the generators.) Of course, one must take into account the ghost contributions to the constraints. Instead, if we choose to work in the light-cone gauge in which all symmetries apart from (6.1) are fixed, then we must show that the rigid super-Poincaré algebra and the residual symmetry algebra,

$$\left\{\varepsilon^{ab}\nabla_a T_b(\xi), \varepsilon^{cd}\nabla_c T_d(\xi')\right\} \approx \partial_a\delta^2(\xi-\xi')\varepsilon^{ab}\nabla_b\left(\varepsilon^{cd}\nabla_c T_d\right)(\xi') \tag{6.2}$$

hold at the quantum level. Here, the ghost contribution to the constraint must be taken into account.

The nontrivial commutators of the super-Poincaré algebra are $[M^{r-}, M^{s-}] = 0$, and $\{\Gamma_-\Gamma_+Q, \Gamma_-\Gamma_+Q\} = \Gamma^+P^-$. The expressions for the generators M^{r-} and $\Gamma_-\Gamma_+Q$ are furnished by eqs. (4.27), (4.28), (5.14) and (5.16)–(5.19).

Any nonclosure in the constraint or super-Poincaré algebra should signal the presence of anomalies in local or global reparametrizations and/or fermionic invariances of the theory. In this context, we note that it is not clear how to write down a lagrangian for massive regulator fields which would respect the fermionic invariance.

As far as the Virasoro-like symmetries of the supermembrane are concerned, although a lot remains to be done, we note that Hoppe [9] has already interpreted the reparametrizations of the bosonic membrane by divergence-free 2-vectors as infinite parameter symmetries which are related to the SU(N) algebra for $N \to \infty$. Note that the closed string analogue of these transformations are the constant σ-translations generated by $\oint T_\sigma \mathrm{d} l^\sigma = \oint (T_{++} - T_{--}) \mathrm{d}\sigma = N - \bar{N} = 0$, where N and $\bar{N}$ are the number operators for the left and right movers.

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Appendix

NOTATION AND CONVENTIONS

$$\text{Signature of } g^{ij} = (-++), \qquad g = \det g_{ij}, \tag{A.1}$$

$$\text{Signature of } \eta_{\mu\nu} = (-,++++++++++), \tag{A.2}$$

$$\varepsilon^{012} = -1, \qquad \frac{1}{\sqrt{-g}} \varepsilon^{ijk} \text{ is a tensor}, \tag{A.3}$$

$$\varepsilon^{ijk} \varepsilon^{mnp} = g\left(g^{im} g^{jn} g^{kp} + 3 \text{ more} \right), \tag{A.4}$$

$$\{ \Gamma^\mu, \Gamma^\nu \} = 2\eta^{\mu\nu}, \qquad \Gamma^{\mu\dagger} = \Gamma^0 \Gamma^\mu \Gamma^0, \tag{A.5}$$

$$\Gamma^{\pm} = \sqrt{\tfrac{1}{2}} (\Gamma^0 \pm \Gamma^{10}), \tag{A.6}$$

$$(\Gamma^+)^2 = (\Gamma^-)^2 = 0, \qquad \{ \Gamma^+, \Gamma^- \} = -2, \tag{A.7}$$

$$\psi^\alpha = C^{\alpha\beta} \psi_\beta, \qquad \psi_\alpha = \psi^\beta C_{\beta\alpha}, \qquad C_{\alpha\beta} = -C_{\beta\alpha}, \qquad C^{\alpha\beta} C_{\beta\gamma} = -\delta^\alpha{}_\gamma, \tag{A.8}$$

$$\bar{\psi} = \psi^\dagger \Gamma_0, \qquad (\bar{\chi} \Gamma^{\nu_1, \dots \nu_n} \lambda)^\dagger = -\bar{\chi} \Gamma^{\nu_1 \cdots \nu_n} \lambda \qquad \text{for any } n, \tag{A.9}$$

$$\bar{\chi} \lambda = \chi_\alpha \lambda^\alpha, \qquad \bar{\chi} \Gamma_\mu \lambda = \chi_\alpha (\Gamma_\mu)^\alpha{}_\beta \lambda^\beta, \qquad \text{etc}. \tag{A.10}$$

(Anti)symmetrizations are with unit strength, e.g. $\Gamma^{\mu\nu} = \frac{1}{2}(\Gamma^\mu \Gamma^\nu - \Gamma^\nu \Gamma^\mu)$. $(\Gamma^\mu)_{\alpha,\beta}, (\Gamma^{\mu\nu})_{\alpha\beta}$ are symmetric, $(\Gamma^{\mu\nu\rho})_{\alpha\beta}$ is antisymmetric.

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